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ABSTRACT

This paper explores the nature and status of the mathematics reform movement (focusing on California's initiatives) in America's public schools, the connection between the reform movement and constructivist epistemology, the development of an assessment tool for measuring the degree of reform present in a secondary mathematics classroom, and the potential for investigating the relationship between the degree of reform and student achievement. Data analysis support the notion that measuring the degree of reform is possible and several instruments have a statistically significant degree of correlation with the expert rating. A discussion of future considerations is included and cautions against the use of rhetoric about reform now that research-based methods are available to measure the degree of reform. (Contains 49 references.) (DDR)

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THE MATHEMATICS REFORM MOVEMENT: ASSESSING THE DEGREE OF REFORM IN SECONDARY MATHEMATICS CLASSROOMS

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THE MATHEMATICS REFORM MOVEMENT: ASSESSING THE DEGREE OF REFORM IN SECONDARY MATHEMATICS CLASSROOMS

The purpose of this paper is to explore: a) the nature and status of the mathematics reform movement in America's public schools, b) the connection between the reform movement (focusing on California initiatives) and constructivist epistemology, c) the development of an assessment tool for measuring the degree of reform present in a secondary mathematics classroom, and d) the potential for investigating the relationship between the degree of reform and student achievement.

Student performance in mathematics at the secondary level has been an issue of great concern to American educators, parents, and government officials, especially in the last 15 years. Declining student scores in mathematics in the United States and poor national performance in comparison to other countries have led to a widespread call for a reform of public education (National Commission on Excellence in Education, 1983; National Research Council, 1989). With the publication of *A Nation at Risk* by the National Commission on Excellence in Education (1983), the *Curriculum and Evaluation Standards by the National Council of Teachers of Mathematics* (NCTM) in 1989, and the California Department of Education's *Mathematics Framework* (1992), a new reform platform was established in California for mathematics education. The new platform included: (a) the identification of the content to be taught into 14 mathematics standards, (b) recommendations about instructional strategies and student activities, and (c) a new underlying philosophy about how learning happens in the mathematics classroom.

Despite these publications, student performance in California has continued to fall behind that of most other states and developed countries (California Department of Education, 1995). Recent results from the Third International Mathematics and Science Survey (TIMSS) indicate that mathematics achievement scores for students in the United States are below levels in other industrialized countries (U.S. Department of Education, 1996). These declining scores have been used by some as a justification for a call to return to a traditional approach to teaching mathematics, focusing on the acquisition of skills and memorization of facts. Others favor the reformed approach, based on the NCTM *Standards* and the California *Mathematics Framework* (1992). California State Superintendent of Public Schools Delaine Eastin charged a Task Force in April 1995 to make recommendations to "do what needs to be done to turn education in mathematics around" (California Department of Education, 1995, p. 1). The California Department of Education issued the California *Mathematics Task Force Report* to the Superintendent on September 13, 1995. Included in the report are the recommendations of the Task Force for improving mathematics achievement in California schools (California Department of Education, 1995, p. 3). These recommendations include a call for:

1. The establishment of clear and specific content and performance standards for mathematics and support of districts and schools to make these standards achievable by all students.

2. The establishment of a stable, coherent, and informative system of assessment for all California students.
3. The guarantee of high quality mathematics instruction by providing adequate time, instructional materials, mathematically powerful teachers, and time for teacher collaboration.
4. The establishment of a management, research and information system to answer basic questions about the operation and effectiveness of mathematics policies and programs, including implementation and effectiveness of the state's *Mathematics Framework*.
5. The identification of responsibilities for the school and the home to support the parents' role in their children's education.

While the Department of Education has initiated activities related to the first two recommendations, the latter three recommendations have received little attention to date. Of particular interest is Recommendation 4, calling for research to improve mathematics education. The state *Mathematics Framework* (1992) describes the new platform for mathematics education, but has no system for determining whether it is effective, or even whether the reforms are being implemented in California classrooms. The Task Force recommended that a new program be established to monitor the implementation and effectiveness of educational policies. The function of such a program would be to undertake systematic observations of mathematics classrooms in action and studies of student achievement, in order to determine which programs and policies are effective.

The purpose of this study was to develop and assess the predictive validity of an instrument measuring the degree to which secondary mathematics classrooms have implemented the policies, strategies, and philosophy of the California *Framework* (1992), and the NCTM *Standards* (1992). The instrument will utilize questionnaire data and videotapes of classroom instruction. The degree of reform will be assessed along the following four areas: (a) teacher beliefs, (b) teacher activities, (c) student activities, and (d) curricular content and methodology.

The Nature Of The Mathematics Reform Movement

The California Framework describes reformed mathematics classrooms by describing the abilities of learners: "Mathematically powerful students think and communicate, drawing on mathematical ideas and using mathematical tools and techniques" (California Department of Education, 1992, p. 20). The four dimensions of mathematical power are then described as mathematical thinking, communication, drawing upon mathematical ideas, and the use of mathematical tools and techniques. These dimensions parallel the process standards in the NCTM *Curriculum and Evaluation Standards* (1989):

1. Mathematics as Problem Solving
2. Mathematics as Communication
3. Mathematics as Reasoning
4. Mathematical Connections

The reformed mathematics described in each of the documents offers guidelines in the arenas of curricular materials, instructional strategies, and student activities. Underlying the recommendations of both documents is a constructivist philosophy of how learning occurs.

Constructivism as a model of learning has become a popular theoretical basis for many educational reform decisions made in Western societies. The tenets of constructivism have served as a basis for the beliefs of many teachers and researchers about the way classrooms should operate. The National Research Council declares, "Research in learning shows that students actually construct their own understanding based on new experiences that enlarge the intellectual framework in which ideas can be created" (1989, p. 6). The California *Mathematics Framework* (1992) affirms the research of Piaget and Vygotsky, stating that students are not passive absorbers of knowledge, but rather constructors of their own understanding of mathematics. The *Framework* stresses that students learn mathematics by using it to make sense of their own experience. This learning is made more powerful when students are able to use it "to achieve purposes that are meaningful to them" (California Department of Education, 1992, p. 33).

The Process of Making Meaning

Analysts of the knowledge construction process have outlined several component activities typically present in developing meaning in mathematics: Encountering a new situation, comparing the new situation with prior knowledge, hypothesis creation and testing, and reflection.

Encountering a new situation. Knowledge construction begins when the learner encounters a new situation (Appleton, 1993). Tobin (1989) calls this first step sense "perception." Data come into the mind through the physical senses. Often this is where most education stops, as many objectivist-oriented educators believe that knowledge has been acquired (Etchberger & Shaw, 1992; Roth, 1993). These data can come in many forms: a new experience encountered, a new problem posed, or a simple observation. Then the data are checked against prior knowledge.

Comparing the new situation with prior knowledge. After the sense perception and analysis the learner may, appropriately or not, assimilate this experience along with some prior knowledge. If the data do not fit into an existing cognitive framework however, a state of perturbation is entered. Several theorists have described this part of the learning cycle as the key experience in learning, calling it by such names as dissonance (Festinger, 1957), disequilibrium (Piaget, 1978), mini-theory testing (Claxton, 1990), or perturbation (von Glasersfeld, 1987). Cobb (1994b) writes, "perturbations that the cognizing subject generates relative to a purpose or goal are posited as the driving force of development" (p. 14). This perturbative stage is where the individual becomes motivated to gather more data and to create and test hypotheses. Fosnot (1989) calls this process disequilibration, following Piaget's terminology, and seeks to activate this sense in her students by selecting new situations which will cause them to feel discomfort.

Hypothesis creation and testing. Whether the new situation was assimilated into a previously acquired knowledge scheme or a new hypothesis was created, the mind investigates the viability of the thinking generated (Fosnot, 1989; von Glasersfeld, 1995a). The learner often concludes that this knowledge is relevant to the task at hand and can be used to arrive at a solution (Davis & Maher, 1990). This action to resolve disequilibrium may involve utilizing multiple approaches (Confrey, 1995).

Reflection. The next stage is an analytical one described by many theorists as reflection (Etchberger & Shaw, 1992; von Glasersfeld, 1987; Wheatley, 1992). The learner is engaged in making sense of the gathered data and his/her representation of the solution process. Reflection is described as distancing oneself from the act of doing and looking at one's own actions. This can be done individually or socially (Cobb, 1994b). Reflection typically has three evaluative components: (a) The interpretation of the problem, (b) a review of the viability of the process used, and (c) justification of successful strategies (Confrey, 1990). Piaget termed this process "reflective abstraction" and it is interesting to note that two alternate definitions of "reflection" are both appropriate to the work done by the learner: reflection as "pondering" and reflection as "bouncing my ideas off of other minds for feedback" (von Glasersfeld, 1995b). During reflection, one constructs schemes of schemes, asking "Will my approach work?" Often the answer is negative and the learner must continue in disequilibrium. Here, or at several other stages, the learner may also elect to opt out of learning (Appleton, 1993).

If the analysis proves that the learner's hypothesis is correct, or illuminates another more efficient hypothesis, the mind then adapts internal cognitive structures to accommodate the new conjecture. Satisfied, the perturbation is eliminated and equilibrium returns.

Because most classrooms today are operating from the assumptions of objectivist or behaviorist theories, determining how to change the classroom to reflect the reform based on constructivist philosophy is an important task. Priority needs to be given to how a student is constructing knowledge, with less emphasis on information being delivered by the instructor.

One note of caution must precede discussion of classroom characteristics that support constructivist theory. Cognizing minds construct knowledge regardless of how experiences are encountered. Whether a student is presented with an authentic experience or a worksheet with 50 arithmetic problems, the student will use the processes of accommodation and assimilation to reestablish equilibrium. Students construct knowledge even in the most authoritarian settings. Constructivism is a theory of how knowledge is constructed, not necessarily a handbook for classroom practice. Research must focus on the nature and quality of the constructions and which classroom environments best contribute to quality knowledge construction, rather than whether or not knowledge construction is occurring (Cobb, 1994a).

Classroom Applications of Constructivist Epistemology

Recommendations for applying constructivist ideas to the classroom are centered around three broad themes: Teacher beliefs and activities, curricular prompts and activities, and student processes and activities.

Reform recommendations about teacher beliefs and activities. Inspection of the literature reveals that many of the guiding principles of the *Framework* (1992) about the activities of the teacher have been informed by researchers of a constructivist-based philosophy of learning. Teacher beliefs, conceptions and personal theories about subject matter, teaching, and learning all affect the creation of a constructivist environment (Ernest, 1995). The quality of student constructions depends in large part on the assumptions and activities of the teacher (Carroll & Balfanz, 1995).

The *Mathematics Framework* (1992) devotes an entire section to the role of the teacher in creating mathematically powerful classroom environments. The optimal atmosphere has students taking responsibility for understanding, doing viable work, managing time resources, and communicating with others. Teachers are to assume the roles of guide and pacer, and need to model what it looks like to be a practicing mathematician.

The *Framework*'s guiding principles for teaching for understanding are as follows:

1. Development of student thinking and understanding.
2. Creating direct personal experiences in which students can verify or alter their own understanding, rather than depend on an authority.
3. Deep understanding takes time; teachers need to provide many varied opportunities to explore a particular concept.
4. Students learn at many levels and rates. Teachers must provide activities that can be understood at many levels.
5. Students need to work with each other in small groups.
6. Periods of confusion are an important part of learning; teachers must resist the temptation to give immediate answers and instead frame questions to further investigation.
7. Teachers must be students of the thinking process that learners are going through.
8. Students must be allowed to reflect upon the ideas found in activities.
9. Teachers should foster a questioning attitude in students.
10. Teachers need to recognize the importance of student verbalization.

(California Department of Education, 1992, pp. 52-53)

Pirie and Kieren (1992) record four underlying tenets of belief necessary to creating a constructivist mathematics environment:

1. The teacher must recognize that particular learning goals will be achieved at differing rates and levels by the students; the teacher must be continually recreating situations which reflect current student constructions.

2. The teacher must recognize that there are multiple pathways to similar mathematical understanding; each child comes to his/her current level of understanding through an uniquely individual pattern of engagement with the learning content.

3. The teacher will be aware that students hold different mathematical understandings.

4. The teacher will know that levels of understanding are never reached once and for all.

Implicit in these tenets is the belief that there is no common mathematical understanding waiting "out there" to be transmitted. Understanding is developed rather than transmitted. Teachers must then encourage the development of metaphors and mathematical representation systems (Brooks & Brooks, 1993; Cobb, Wood, Yackel, & Perlwitz, 1992).

A critical activity of the teacher in the constructivist environment is creating situations in which students actively participate in activities that encourage them to make their own individual constructions (Wood, 1995). Teachers can encourage these activities by asking thoughtful, open-ended questions (Brooks & Brooks, 1993) and by placing the focus of activity on student interaction with the problem, not teacher presentation of a skill (Serrano & Stone, 1995).

In an environment marked by constructivist philosophy, the teacher must seek to understand and value the students' point of view (Confrey, 1990; Ernest, 1995; Wood, 1995). Confrey suggested that throughout the learning process teachers must build models of students' understanding of mathematics (Confrey, 1990). She described these models as "case studies" for each student. The perspective that a student brings to an encounter is seen as an instructional entry point in the learning experience. Thus, the process of learning is not centered around having the students focus on what the teacher knows, but rather the teacher focusing on what the student knows and creating experiences which will extend and challenge the student. Asking students to reflect on their thinking, or elaborate on their solution to a problem, allows the teacher to make assessments about how students view a situation. Rather than focusing on "errors in thinking," teachers see student approaches as an indication of current understanding (von Glasersfeld, 1995a). Focusing on student points of view also enables the teacher to employ another principle, adapting curriculum to address student suppositions. Learning is enhanced when the demands of the curriculum have a direct relationship to student suppositions (Brooks & Brooks, 1993; Carroll & Balfanz, 1995).

In the typical mathematics classroom created to foster knowledge construction, many students will each be building their own representation of the problem and its solution. Processing the problem posed will not rely on teacher prescription, but on several of the representations invented by students (Jonassen, 1991). Teachers will need to build much more time into problem analysis and encourage multiple representations and explanations (Merseth, 1993).

Brooks and Brooks (1993) believe that assessing student learning in the context of teaching is imperative to a constructivist environment. Rather than simply monitoring the body of facts that a student has committed to recall, assessment embodies the much broader activities of viewing how the student is constructing knowledge.

Reform recommendations about written curriculum. In addition to teacher practices, the curriculum itself can reflect reformed mathematics principles. The *Mathematics Framework* (1992) recommends that written curriculum be created as an instructional unit. A powerful instructional unit shows much more depth than a collection of lessons and activities. The instructional unit described by the *Framework* is coherent, with a clearly formulated purpose. All activities are related to a primary goal, and developed in an engaging, meaningful context:

> Each coherent piece interweaves strands, ensuring breadth, and each deepens one or more unifying ideas. In each, students will do mathematics and be responsible for the thinking. They will investigate rich and complex problem situations, conjecture, explore ideas, make connections among mathematical ideas, and generalize their findings. (California Department of Education, 1992, pp. 90-91)

Curriculum units are recommended to be about 6 weeks long for secondary students, focusing on large assignments (multi-day), open-ended problems, and exercises which lead toward developing the main idea in the unit. Again, the *Framework*'s view of curriculum that supports powerful learning of mathematics can be found in the research on constructivist learning.

Curricula created to foster knowledge construction require instructors to pose problems of emerging relevance (Brooks & Brooks, 1993). Students do not need to have prior interest or complete autonomy in choosing topics of study for the relevance principle to be met. Relevance can emerge as teacher and students interact with a problem that demands the generation of a hypothesis, has multiple approaches and is complex. Relevance is considered to be an important factor in student motivation according to Keller's ARCS model of motivation (Bohlin & Bohlin, 1995). Furthermore, students who are experiencing perturbance with respect to a new situation will find the problem solving process to be relevant to their own understanding. Various researchers have stressed the need for relevant curricula which will encourage teacher use of raw data and primary sources (Brooks & Brooks, 1993; Roth, 1993).

Another principle of a constructivist curriculum is to structure learning around primary concepts. Brooks and Brooks (1993) state that students are more engaged when problems are presented holistically than when component skills are taught in isolation from each other. Constructed knowledge must revolve around an essential idea. Other related characteristics of the principle of presenting big ideas are their relevance and openness for multiple approaches.

Constructivist curricula employ discovery learning as a typical approach to investigation of topics. Davis (1990) names discovery-oriented curricula as one of the effective practices of the "New Math" movement in the late 1950s through the 1970s. In discovery learning, each activity is planned to guide the student to draw his/her own conclusions, to make generalizations and to develop the mathematical concepts necessary to solve the larger problem. Students are required to communicate their ideas in the micro-society of the classroom, presenting, explaining and justifying the mathematics that they have developed.

Reform recommendations about student activities. In addition to teacher behaviors and curricular formats, there are several activities unique to the learners themselves that are indicative of a reformed mathematics classroom. One of these activities revolves around the concept of student autonomy and initiative (Brooks & Brooks, 1993; Roth, 1993; Taylor & Fraser, 1991). If students create their own interpretations of mathematics regardless of the environment (Cobb, 1994b; Wood, 1995), then it stands to reason that they should exercise control over decisions about their own learning process. Case studies of teachers who exhibit constructivist ideas reveal many occurrences of student autonomy and initiative (Clarke, 1995; Confrey, 1990; Roth, 1993; Wood, Cobb, & Yackel, 1995). Measurement tools of the constructivist environment include scales for student decision making with respect to all classroom practices (Taylor & Fraser, 1991). Case studies of teachers in training reveal that student self-regulation is actually a component of constructivism that teachers discover through teaching experiences (Fosnot, 1989). These researchers all indicate that student self-assessment and decision making about how to proceed is more effective than teacher advice on "how to do it."

An equally important student activity is the use of student dialogue in the reflection process (Brooks & Brooks, 1993; Cobb, 1994b). Instruction is interactive, involving both student-to-student and student-to teacher-dialogue (Cobb, Wood, & Yackel, 1990; Confrey, 1990). Driver (1995) writes that discussion with peers is an important element in knowledge construction. Discussion provides an arena for making implicit ideas explicit. It allows individuals to clarify their thinking, and offers the opportunity for students to build from each other's ideas a deeper understanding of the concepts involved.

Through dialogue and student decision making, students have the opportunity to examine the viability of their personal constructions. Students therefore have the opportunity to adapt their constructions when they see misconceptions. Several researchers have attested to the tenacity with which children hold to naive views of problem situations (Bodner, 1986; Merseth, 1993; von Glasersfeld, 1995a). Case studies reveal that teacher-provided information contrary to these misconceptions does little to alter thinking. Von Glasersfeld (1995a)writes:

> As long as the counterexamples provided by the teacher are taken from areas that lie outside the students' field of experience, they are unlikely to lead to a change in the students' thinking. Only when students can be led to see as their own a problem in which their approach is manifestly inadequate will there be any incentive for them to change it. (p. 15)

Dialogue is necessary in other important components of the learning process: (a) Elaboration of initial responses (Brooks & Brooks, 1993), (b) Cooperative learning (Etchberger & Shaw, 1992), and (c) Consensus building among student groups (Etchberger & Shaw, 1992).

The Purpose of Mathematical Constructions

The goal of student, teacher, and curricular activities is powerful student mathematical construction. Powerful constructions are characterized by Confrey (1990) as having the following characteristics:

1. A structure with a measure of internal consistency.
2. Integration across a variety of concepts.
3. Convergence among multiple forms and contexts of representation.
4. An ability to be reflected on and described.
5. Historic continuity.
6. Ties into various symbol systems.
7. Agreement with experts.
8. A potential to act as a tool for further constructions.
9. A guide for future actions.
10. An ability to be justified and defended.

Measuring the Degree of Reform in Mathematics Classes

Reformed instruction and constructivist practices have become benchmarks for quality reformed mathematics learning among leading educators and theoreticians. However, very little has been done to quantify the effects of a reformed environment on student performance. There are several obstacles to be overcome before quality research can be accomplished regarding the effectiveness of these environments.

If constructivist pedagogy is to be used as a basis for defining effective instruction as an independent variable, then a coherent, well-planned instrument must be developed to discriminate environments as either mathematically reformed or traditional. None of the instruments currently available incorporate all four elements described here: curriculum, teacher beliefs, teacher activities, and student activities. Any instrument to be developed must be tested before classrooms can be validly and reliably identified along a reformed/traditional continuum.

Finally, the issue of how to properly assess students who learn in a reformed environment has also been raised. Along with the call for reform in mathematics education, there has been a call for reform in assessment. Many educators believe that traditional textbook and nationally-normed tests have served as profound barriers to educational reform. Because fill-in-the-blank and multiple-choice tests do not provide meaningful measures of higher-order thinking and reasoning skills, these skills are not valued in the typical classroom (Merseth, 1993).

Determining what types of assessment tools best measure student achievement is a hotly debated educational and political issue. Although authentic assessment instruments are being developed, there is some doubt about the validity and reliability of these instruments. It does not seem prudent or appropriate however, to rely on standard multiple-choice assessments to determine the effectiveness of knowledge construction.

Methodology

What follows in this paper is the presentation and analysis of an instrument to measure specifically where secondary mathematics classrooms fall on a traditional/reformed continuum. The instrument utilizes a video analysis inventory to measure observed behaviors, and a questionnaire to measure teacher beliefs about the classroom environment.

The Instrument

The two-part measurement instrument which was developed to be used in the study consists of a teacher beliefs questionnaire, and a video analysis inventory, named the Secondary Mathematics Classroom Reform Inventory (SMCRI), which is divided into three subscales. The three subscales of the SMCRI, and the Teacher Beliefs Questionnaire comprise the four independent variables related to the areas of classroom characteristics described in the literature review.

1. The Teacher Beliefs Questionnaire contains 31 items probing teachers' beliefs about how learning should take place and actual instructional practices reported by the teacher. The questions were designed to avoid influencing the teacher either toward a reform perspective or a traditional philosophy of instruction. A summary score was given for each teacher, indicating placement along the continuum from reformed to traditional philosophy and practice. The response scales are Likert five-point scales, with a score of 1 indicating little or no reform, and a score of 5 indicating a high degree of reform.
2. The Secondary Mathematics Classroom Reform Inventory (SMCRI) was developed to rate how fully classroom activities and prompts are occurring in the classroom. The SMCRI was used by the classroom/video observer, in this case the researcher. The response scale for all items in the SMCRI is a five-point Likert scale, with 1 = Completely absent and 5 = Fully emphasized. The three subscales on the SMCRI are:

A. The SMCRI Teacher Activities Subscale, containing 14 items assessing the nature of what teachers do to enable student processing, communication, and reflection.

B. The SMCRI Curricular Activities Subscale, containing 9 items indicating degree of curricular complexity, and identifying the sources and prompts used.

C. The SMCRI Student Activities Subscale, containing 9 items observing student autonomy, communication, and cognitive functions.

A fifth variable used in the study was the expert holistic rating of the degree to which the classroom exhibits the characteristics of reform. This rating serves as the dependent variable in the study. For this variable, overall holistic ratings independent of the SMCRI were provided by an expert in reformed mathematics education for the videotapes of the mathematics lesson. This holistic rating used the following 5-point rubric:

1 = No evidence of reformed curriculum and instructional practices
2 = Little evidence of reformed curriculum and instructional practices
3 = Reformed curriculum and instructional practices are somewhat evident

4 = Reformed curriculum and instructional practices are largely evident
5 = Reformed curriculum and instructional practices are completely evident

Procedure

Teachers from two high schools in Central California were invited to participate in the study. They were informed that the study involved observation of various teacher and classroom characteristics, and that the study also sought to compare teacher beliefs with instructional practices. Two of the 17 teachers who were asked to participate declined. Teachers were asked to select a class period and course that best represents their teaching style. These 15 mathematics classrooms were videotaped for one period during the month of February in the 1995-96 school year. The Teacher Beliefs Questionnaire was filled out by the same teachers approximately 2 weeks prior to the videotaping and returned to the researcher.

The classrooms reflect various socioeconomic communities and diverse student ethnic populations. Both high schools are populated with students ranging from low- to middle-socioeconomic status. Both schools have approximately the same socio-ethnic populations: 45% Hispanic, 30% Euro-American, 10% African American, and 15% Asian-American. Both schools receive Tittle 1 funding because a certain percentage of the student population demonstrates standardized test scores below the 45% percentile. The courses videotaped include: Math A, Algebra 1, Geometry, Algebra 2, Interactive Mathematics 1 and Interactive Mathematics 3. The Interactive Mathematics Program is a National Science Foundation sponsored curriculum project intended to reflect the NCTM *Standards*.

The videotapes focused on the teacher vocalizations, student vocalizations, and various other activities in the classroom. The videos were taped by high school students. The tapes focus on the instructor during all lecture periods, and on particular groupings of students who were either engaged in the activity, or working directly with the teacher during student activities. In addition, a copy of the written prompt/text for the lesson was provided to the researcher. Evaluations along the Curricular Scale utilized both the written prompt and the activities of both teachers and students.

The videos were observed by the researcher using the SMCRI, then by mathematics-educator-experts who had received training in the teaching of reformed mathematics for at least 100 hours over the past five years. The training included either participation in the San Joaquin Valley Mathematics Project or in inservicing for a particular reformed mathematics curriculum. Each of the 15 videotapes was viewed and rated by three experts. Each expert viewed a different combination of tapes. Interrater reliability was established by determining the percent of ratings given which match at least one other expert rating. If the percent agreement met or exceeded 67% (four of six ratings), then the expert was considered reliable. Once interrater reliability was established, the mean of the ratings of the reliable experts was calculated and used as the dependent variable.

Statistical Analyses

The usefulness of the instrument was determined by analyzing its predictive validity. Means and sample standard deviations were computed for each of the individual items on the instrument for descriptive purposes. Means and summary scores were computed for each of the teachers on each subscale in order to determine if the subscales produced a range of scores. Correlations were computed using the summary score for each of the independent variables, the sum of the video subscales, the sum of the video and questionnaire subscales, and the expert rating. The Pearson correlation coefficient was used to determine the degree of interrelatedness between independent variables, and the degree of relatedness between single subscales and the expert rating. Multiple linear regression was performed to determine the predictive validity of the instrument in determining the degree of reform evident in a particular classroom setting.

Results

Tables 1 and 2 summarize the data collected from the Teacher Beliefs Questionnaire and the Secondary Mathematics Classroom Reform Inventory. The data from the questionnaire is presented in Table 1, showing means and teacher summary scores for the subscale. The video data is presented in Table 2, showing each of the SMCRI video subscales, and teacher summary scores for each of the subscales. The means provide information about the center of the responses on the five-point response scale, while the standard deviation offers insight into the variability seen in the responses from the 15 instructors. All 15 teachers responded to each of the items in the teacher beliefs questionnaire, and data was collected for each teacher for every item on the classroom inventory form. Therefore, $n = 15$ in all tables.

Teacher Beliefs Questionnaire and SMCRI Summary Scores

Teacher Beliefs Questionnaire. The teacher questionnaire responses were added together, and a mean was computed for each teacher. Table 1 contains the raw score total and mean for each of the teachers. The teachers are identified simply as teacher #1 through #15. The mean raw score for the teachers in the study was 113, with a standard deviation of 20.1. The mean of the teachers' averages was 3.64, with a standard deviation of 0.65. Eleven of 15 (73%) teachers in this study self-report their teaching methods and philosophy to be between 2 and 4, which is somewhere between partially reformed and largely reformed on the traditional/reformed continuum.

SMCRI. One research question to be addressed in the study refers to the ability of the SMCRI to indicate an acceptable range of summary scores for each of the subscales. Table 2 presents the teacher summary score for each of the three video subscales, as well as the mean score on that subscale. The Teacher Activities subscale scores ranged from 1.0 to 4.1, with a mean of 2.39. The Curricular Activities subscale exhibited a range from 1.0 to 5.0, with a mean of 3.15.

Table 1

Summary Scores and Means by Teacher on the Teacher Beliefs Questionnaire

Teacher Questionnaire Totals by Teacher	Sum of Item Responses	Mean
Teacher #1	107	3.5
Teacher #2	116	3.7
Teacher #3	83	2.7
Teacher #4	120	3.9
Teacher #5	126	4.1
Teacher #6	114	3.7
Teacher #7	125	4.0
Teacher #8	87	2.8
Teacher #9	88	2.8
Teacher #10	90	2.9
Teacher #11	96	3.1
Teacher #12	144	4.6
Teacher #13	151	4.9
Teacher #14	143	4.6
Teacher #15	121	3.9

Table 2

Sums and Means on the Secondary Mathematics Classroom Reform Inventory by Each Teacher on the Teacher, Curricular, and Student Subscales

Teacher Scores	Teacher		Curricular		Student	
	Sum	Mean	Sum	Mean	Sum	Mean
Teacher #1	30	2.1	29	3.2	22	2.4
Teacher #2	53	3.8	44	4.9	32	3.6
Teacher #3	28	2.0	10	1.1	13	1.4
Teacher #4	24	1.7	21	2.3	12	1.3
Teacher #5	41	2.9	38	4.2	31	3.4
Teacher #6	38	2.7	40	4.4	22	2.4
Teacher #7	24	1.7	22	2.4	25	2.8
Teacher #8	38	2.7	31	3.4	25	2.8
Teacher #9	17	1.2	12	1.3	11	1.2
Teacher #10	14	1.0	11	1.2	9	1.0
Teacher #11	26	1.9	9	1.0	12	1.3
Teacher #12	48	3.4	42	4.7	30	3.3
Teacher #13	32	2.3	44	4.0	20	2.2
Teacher #14	58	4.1	45	5.0	36	4.0
Teacher #15	31	2.2	27	3.0	12	1.3

The Student Activities subscale displayed a range from 1.0 to 4.0, with a mean of 2.31. The data suggest that this sample produced a wide range along the possible continuum between traditional and reformed classroom environments.

Correlation of Variables

Table 3 presents results of a correlational analysis which was conducted between the four independent variables, the video summary score, the video plus questionnaire summary score, and the expert mean for each of the 15 subjects. All of the combinations of variables had a high degree of correlation with the lowest being $r = .612$ and all of the correlations were significant at the $p < .015$ level. The highest correlation was between the Teacher Activity subscale and the summary score for the questionnaire and the video subscales combined, $r = .958$. This value is inflated because the Teacher Activity subscale score is included in this particular summary score as well. Each instrument subscale correlated highly with the expert rating as well, the lowest $r = .757$, and all were significant at the $p < .001$ level

Regression Analysis

Multiple linear regression analysis was performed using the summary score for the Teacher Beliefs Questionnaire, and each of the three SMCRI video subscales as the four independent variables. The dependent variable was the mean of the three reliable expert ratings. Table 4 summarizes the analysis.

The resulting predictive equation is as follows:

$$Y' = -1.569 + .016(\text{Tchr. Q.}) - .031(\text{Tchr. Act.}) + .026(\text{Curr. Act.}) + .138(\text{St. Act.})$$

The F-ratio value for the analysis ($F_{4,10} = 26.15$, $p < .00001$). The multiple $R = .95538$. For this set of data, the R^2 is .91 and the adjusted R^2 is .88, so the regression equation is able to account for approximately 88% of the variance in the expert ratings.

Table 4 reports also that the student activities subscale was the only variable which could singularly account for the variability in the data ($\beta = .815$, t-value = 3.868, t-value significant at $p = .0031$). This does not necessarily imply that the other variables are not contributing factors, because each of the independent variables has a high degree of intercorrelation.

Limitations

There are several limitations in this study. Many of these limitations can be accurately addressed by further testing and analysis of the Teacher Beliefs Questionnaire and Secondary Mathematics Classroom Reform Inventory.

Table 3

Intercorrelations Between Subscales for Secondary Mathematics Classroom Reform Inventory, Teacher Questionnaire, and Expert Ratings

Subscale	1	2	3	4	5	6	7
1. Expert Ratings	–	855 ($p<.0001$)	.921 (.0001)	.792 (.0001)	.757 (.001)	.895 (.0001)	.907 (.0001)
2. Curricular Activities SMCRI		–	.825 (.0001)	.841 (.0001)	.751 (.001)	.947 (.0001)	.940 (.0001)
3. Student Activities SMCRI			–	.882 (.0001)	.635 (.011)	.938 (.0001)	.887 (.0001)
4. Teacher Activities SMCRI				–	.612 (.015)	.958 (.0001)	.891 (.0001)
5. Questionnaire					–	.708 (.003)	.881 (.0001)
6. SMCRI Summary						–	.958 (.0001)
7. TQ + SMCRI Sum.							–

Table 4

Summary of Simultaneous Regression Analysis for Variables Predicting the Expert Rating of the Degree of Reform in the Classroom (n = 15)

Variable	B	β	t	Sig. t
Teacher Beliefs Questionnaire	0.017	.222	1.557	.151
Teacher Activities	-0.031	-.259	-1.177	.266
Curricular Activities	0.026	.234	1.099	.298
Student Activities	0.138	.815	3.868	.003

Note. Multiple R = .95538; R^2 = .912275; Adjusted R^2 = .87786

No attempt was made to select a representative sample of teachers to participate in the study. The objective was to purposefully sample in order to maximize variability, so a range of classroom environments along the traditional/reformed continuum was necessary. Consequently, it is unclear whether the instrument would accurately discriminate among classrooms which are narrowly distributed along the continuum.

Because the instrument was not tested on a sample representative of the population of secondary California mathematics classrooms, no statements can be made about the general degree of reform evident in the state or the nation today. Many of the questions of the Task Force concerning the degree to which the California *Mathematics Framework*'s (California Department of Education, 1992) policies have been implemented would need to be answered by applying the instrument to a representative sample of classrooms.

Another limitation of the study involved the testing of only 15 mathematics teachers and classrooms. Statistics performed on this sample may not yield the same conclusions when a larger sample is included in the study. A larger sample size of 30+ would offer a better perspective from which to study these results.

The limited number of classroom participants also make it difficult to interpret the standardized Beta weights obtained in the statistical analysis. However, standardized Beta weights are inherently unstable, and increasing the number of participants is not likely to allow for their confident interpretation.

There is a high degree of correlation between each pair of variables compared in Table 3. This suggests that multicollinearity between all of the independent variables exists that would impair the ability of any one variable to add unique variance to the R^2 obtained in the regression analysis. While the Student Activities subscale displayed a significant weight in the regression equation, it is likely that any combination of the other subscales could have predicted the expert rating nearly as well, had Student Activities been omitted from the analysis.

Apart from the means and sample standard deviations found for each of the items, no attempt was made to analyze the usefulness of each of the individual items in the Teacher Beliefs Questionnaire or Secondary Mathematics Classroom Reform Inventory form, or to determine whether they would load along one of the subscales of the video analysis to form a validated construct. Performing a factor analysis was not possible because of the limited number of participating classrooms. Such a factor analysis could focus the instrument on the specific constructs which can discriminate the degree of reform in the classroom.

Some of the experts felt that making a decision about the degree of reform evident based on one videotape of one class period was less than desirable. Many of the behaviors which might typically be evident in a particular classroom over the course of several lessons may have been omitted on the day that the videotaping was done. In addition, one of the characteristics of the reformed classroom is multi-day activities. Several of the classrooms observed appeared to be in the middle of solving a complex, multi-day problem. During such a sequence of class periods, observing only one of them is necessarily limiting the nature of the activities exhibited in the sequence.

The question of whether a representative sample of a particular classroom was obtained by just one "snapshot" is also affected by the presence of the video camera. It can be surmised that the presence of the camera affected the classroom environment to some degree in most cases. Many of the videos contained footage in which students were initially disconcerted by the camera, or asked the instructor questions about the videotaping that was being done.

Several video items were scored as "1" meaning, "not evident at all," because the activity in question was not observed. However, the activity in question could have been inferred as normally evident from the general demeanor of the classroom. These item scores may have taken away from the summary score for that subscale simply because they are items not typically present each day. For example, item #14 on the Teacher Activities Subscale judges "How fully does the classroom instructor conduct some form of assessment during a learning activity." While the teacher may typically conduct assessment in the context of learning, this may simply not have been the appropriate portion of the activity to conduct an assessment.

Finally, only the researcher used the Secondary Mathematics Classroom Reform Inventory form. Therefore, reliability among users of the inventory was never addressed. Complications concerning reliability will arise if and when other educators or non-experts use the instrument. The instrument's reliability when used by others will need to be investigated to further analyze the instrument's effectiveness.

Discussion

The combination of the Teacher Beliefs Questionnaire score with the three subscales of the video analysis yielded a highly predictive regression equation. This analysis supports the notion that measuring the degree of reform evident in a secondary mathematics classroom is possible.

Because the scales used in the Teacher Beliefs Questionnaire, the SMCRI and the expert ratings are all Likert-type 5 point, with a "1" generally representing "little or no evidence" and a "5" representing "fully evident," comparisons can be made between teacher self-reports and independent ratings. The mean of all teachers on the Teacher Beliefs Questionnaire was higher than the mean for any of the other variables, including the expert ratings, the possibly indicating that teachers are over-reporting where they fall into the traditional/reformed continuum. If this were the case, there would be a discrepancy between teacher beliefs and behavior in the classroom which would merit further investigation. However, it is equally likely statistically that the experts and the SMCRI are underreporting the degree of reform in the classroom. While teachers may have over-reported their degree of reform in the questionnaire, the high correlation between the questionnaire summary score and the expert rating ($r = .757$, $p <= .001$) suggests that they accurately rank themselves relative to their colleagues. The summary score for the Teacher Beliefs Questionnaire could stand alone as a valid independent variable in predicting the degree of reform in the classroom. This high correlation between the questionnaire score and the expert rating suggests that a much less cost intensive approach than a video analysis may be effective in determining placement on the

traditional/reformed continuum for a classroom environment. This area of investigation could yield a cost-effective approach to examining reform.

Each of the other three independent variables from the SMCRI also had a statistically significant degree of correlation with the expert rating. Each of them therefore, could stand alone as a valid predictor of the expert rating. While none of them could account for the amount of variability that the combination of questionnaire and video subscales offered, they are each indicators of the degree of reform evident in the classroom. In fact, each variable alone would have accounted for a large portion of the variability in the expert ratings: (a) Teacher Beliefs Questionnaire, $R^2 = .5723$, (b) SMCRI-Teacher Activity subscale, $R^2 = .6274$, (c) SMCRI-Curricular Activity subscale, $R^2 = .7312$, and (d) SMCRI-Student Activity subscale, $R^2 = .8473$.

The Student Activities Subscale of the SMCRI offered the highest individual correlation with the expert rating, and appeared in the regression analysis with statistically significant standardized Beta weight. This might suggest that while other activities and beliefs may be important in determining reform, ultimately it is what the students are doing that offers the best indicator of the classroom environment. The correlation also suggests that a colloquial version of the SMCRI, including only the Student Activities subscale, could provide an equally valid analysis of the degree of reform in the classroom.

Future Considerations

The California Mathematics Task Force Report (California Department of Education, 1995) recommended that a research system be established to study the implementation and effectiveness of the state's *Mathematics Framework*. Task Force members were "astonished . . . to discover that the state has no means to determine the effectiveness of these state policies. The state has collected data on student achievement, but not on the processes of teaching and learning that lead to student achievement" (California Department of Education, 1995, p.14). Currently the debate over whether the reforms of the *Framework* are producing higher achievement is alive and well. Those in favor of a traditional approach to mathematics education claim that the reform movement has failed. Those in favor of the reforms claim that the reforms of the *Framework* have yet to be implemented in most classrooms, and that low achievement scores are therefore reflective of the traditional approach to teaching and learning. Indeed, the TIMSS Report found that although 8th grade teachers reported familiarity with the NCTM Standards, their classroom practices were in general not those of a reformed classroom (U.S. Department of Education, 1996). The debate has been driven by various political perspectives, still with very little substantiating evidence for either side. Yet evidence of the implementation and effectiveness of the reforms of the *Framework* is what is needed.

The Teacher Beliefs Questionnaire and Secondary Mathematics Classroom Reform Inventory provide the vehicle for determining the degree of reform evident in a classroom. The instrument can potentially be paired with student achievement data in order to determine if the practices, content, and strategies of the *Framework* are producing classrooms showing higher student achievement.

The *Mathematics Framework for California Public Schools* is currently undergoing some revision. One of the charges of the Task Force to the Superintendent of Public Schools was to establish rigorous content standards for K-12 mathematics. If significant modifications in the *Framework* are made as a result of this review process, then some changes in the measurement instrument may need to be made as well. "Reform" is always being redefined, as its definition is contingent on what is considered to be "accepted practice."

As stated in the literature review, the methods by which student achievement is measured is a debated issue. Many questions arise concerning how student achievement should be measured. Some educators believe that long-time standard indicators of achievement should be used, such as standardized tests and college entrance exams. The recent push to include authentic assessment tasks such as portfolio assessment, open-ended questions, performance-based assessment, and CLAS-type assessments represents another approach to assessment. The type of assessments used will likely be based in part upon the content standards developed by the State Superintendent of Public Schools.

Finally, if a widespread analysis of the number of classrooms exhibiting largely reformed environments were performed and few were found, what are the implications for the state? Questions should be asked about the readiness of mathematics instructors to teach in a reformed manner, and the role and influence of the reform delivery systems. If teachers are over-reporting their degree of reform, then that is an indicator that either they are in need of more information about what are the specific characteristics of a reformed classroom, or they need more practice in self-assessment. Perhaps more attention needs to be paid to the type of ongoing inservice training that California teachers are asked to do.

In conclusion, the pieces are in place for an authentic, research-based look at reform and secondary mathematics student achievement in California. Rhetoric about whether traditional or reformed approaches to teaching are effective can now be replaced by a meaningful research-based look at which classrooms are traditional, and which are reformed. Then studies comparing teaching methods and student achievement can be undertaken to provide an accurate picture of the condition of mathematics education in California. Perhaps then we can each cast aside our naive assumptions and construct a model of the learning of mathematics that can be useful for every student in California's public schools.

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